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Research Announcement:

A HOMOGENEOUS NONBIHOMOGENEOUS CONTINUUM

by

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Background

A space X is called homogeneous if for every two points p and q in X there is a homeomorphism h: X → X such that h(p) = q. A space X is called bihomogeneous if for every two points p and q in X there is a homeomorphism h: X → X such that h(p) = q and h(q) = p. Around 1920 B. Knaster asked whether there exists a homogeneous topological space which is not bihomogeneous. C. Kuratowski [3] provided first such example. All examples listed in this announcement are connected, homogeneous, metric spaces that are not bihomogeneous. The existing (non-compact) examples are as follows:

(i) Kuratowski's example, which is a connected subset of a composant of Knaster's indecomposable continuum. This is a connected, not locally compact, not locally connected, one-dimensional subset of the plane. Similar examples, although not planar, can be obtained by taking certain subsets of a composant of a solenoid.

(ii) A locally compact, two-dimensional example described by H. Cook [1]. This example is a subset of the hyperspace of subcontinua of the pseudo-arc.

In 1930, D. van Danzig [4] restated Knaster's problem for compact spaces. The following answers van Danzig's question:

A Compact Example.

There exist a seven-dimensional, locally connected, compact example (see [2]).

Homogeneous spaces which are locally Cartesian products with at least one of the Cartesian factors homeomorphic to the Menger curve admit few homeomorphisms. This property is used in constructing the above example. First, a homogeneous continuum, which is locally a product of two Menger curves and an interval, is constructed. This continuum is a union of pairwise disjoint simple closed curves, each of which has a preassigned orientation. Homeomorphisms can only rearrange these simple closed curves, and are either orientation preserving on all of them, or orientation reversing on all of them.

The example is obtained by replacing each of the simple closed curves with a special manifold, and hence restricting the class of homeomorphisms even further. The space remains homogeneous. The manifold is chosen in such a way that certain homeomorphisms are not allowed. In particular, there exist points which cannot be interchanged by a homeomorphism.

In addition, the example has the following properties: Every point has a neighborhood which is a product of two Menger curves and the Euclidean space E^5. Every point p has a neighborhood U such that p can be interchanged with every point $q \in U$ by an ambient homeomorphism. There are compact and there are noncompact covering spaces of the example which are bihomogeneous. If n is an integer, then some of the n-fold covering spaces are not bihomogeneous. There is a noncompact covering space which is not bihomogeneous.

References

[1] H. Cook, *A locally compact, homogeneous metric space which is not bihomogeneous*, Top. Proc. 11.1(1986), 25-27.

[2] K. Kuperberg, *On the bihomogeneity problem of Knaster*, Trans. Amer. Math. Soc., to appear.

[3] C. Kuratowski, *Un probleme sur les ensembles homogènes*, Fund. Math. 3(1922), 14-19.

[4] D. van Danzig, *Ueber topologish homogene Kontinua*, Fund. Math., 15(1930), 102-125.

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