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by

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**CONTINUOUS EXTENSIONS OF FUNCTIONS
DEFINED ON SUBSETS OF PRODUCTS
WITH THE κ -BOX TOPOLOGY**

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ABSTRACT. Consider these results: (a) [N. Noble, Proc. Amer. Math.Soc. **31** (1972), 613–614] *every G_δ -dense subspace in a product of separable metric spaces is C -embedded*; (b) [Milton Don Ulmer, Ph.D. Dissertation. Wesleyan University (Middletown, CT, USA), 1970 and Pacific J. Math. **46** (1973), 591–602] *every Σ -product in a product of first-countable spaces is C -embedded*; (c) [R. Pol and E. Puzio-Pol, Fund. Math. **93** (1976), no. 1, 57–69; also A. V. Arhangel'skii, Topology Proc. **25** (2000), Summer, 383–416, as corollaries of more general theorems] *every dense subset of a product of completely regular, first-countable spaces is C -embedded in its G_δ -closure*.

The present paper continues the first two authors' earlier initiative [Topology Appl. **159** (2012), 2331–2337], which already generalized those cited results in several ways simultaneously, e.g., κ -box topology on the product spaces; relaxed separation properties on both the domain and the range spaces). Now the authors show:

Let $\kappa \leq \alpha$ satisfy $\lambda < \kappa$, $\beta \leq \alpha \Rightarrow \beta^\lambda \leq \alpha$; let Y be dense in an open subset U of a κ -box product $(\prod_{i \in I} X_i)_\kappa$ with each X_i a T_1 -space; let $q \in X_I \setminus Y$ have the property that for each $J \in [I]^{\leq \alpha}$ there is $y \in Y$ such that $y_J = q_J$; let Z be a regular space with a $\overline{G_{\alpha^+}}$ -diagonal. Suppose that for each $i \in I$ either $\chi(q_i, X_i) \leq \alpha$ or each intersection of κ -many neighborhoods of q_i is another such neighborhood. Then every continuous $f : Y \rightarrow Z$ extends continuously over $Y \cup \{q\}$.

Several corollaries and consequences are given.

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Key words and phrases. C -embedded subspace, G_δ -dense subset, G_κ -diagonal, $\overline{G_\kappa}$ -diagonal, G_κ -set, $\overline{G_\kappa}$ -set, κ -box topology, $P(\kappa)$ -point, $P(\kappa)$ -space, product space, Σ -product.

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