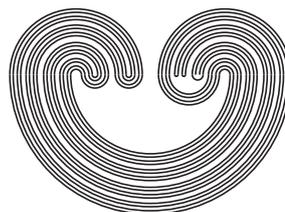


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# TOPOLOGY PROCEEDINGS



Volume 43, 2014

Pages 29–36

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<http://topology.auburn.edu/tp/>

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by

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Electronically published on April 29, 2013

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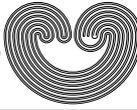
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Department of Mathematics & Statistics  
Auburn University, Alabama 36849, USA

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**ISSN:** 0146-4124

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## ON COMPACT SETS IN $C_b(X)$

JUAN CARLOS FERRANDO

**ABSTRACT.** Let  $X$  be a Tychonoff space and  $C_p(X)$  and  $C_b(X)$  denote the space  $C(X)$  of all real-valued continuous functions on  $X$  provided with the pointwise convergence and the compact-bounded topology, respectively. Let us call an unbounded subspace  $\Sigma$  of  $\mathbb{N}^{\mathbb{N}}$   $\sigma$ -complete if each bounded sequence  $\{\alpha_n\}_{n=1}^{\infty}$  in  $\Sigma$  verifies that  $\sup_{n \in \mathbb{N}} \alpha_n \in \Sigma$ , and let us call  $\sigma$ -complete every family  $\{A_\alpha : \alpha \in \Sigma\}$  of subsets of  $X$  such that  $\Sigma$  is  $\sigma$ -complete and  $A_\alpha \subseteq A_\beta$  whenever  $\alpha \leq \beta$ . We show that if there is a dense subspace of  $X$  covered by a  $\sigma$ -complete family consisting of bounded sets then  $C_p(X)$  is angelic. This is used to prove our main theorem, which asserts that if  $X$  has a dense subspace  $Y$  covered by a  $\sigma$ -complete family consisting of  $Y$ -bounded sets, then  $C_b(X)$  is angelic and every compact set in  $C_b(X)$  is metrizable.

### 1. PRELIMINARIES

Let us start by recalling that a subset  $A$  of a topological space  $X$  is called (functionally) *bounded* [1, Chapter 0] if  $f(A)$  is a bounded set in  $\mathbb{R}$  for every real-valued continuous function  $f$  on  $X$ . On the other hand, a subset  $B$  of a topological vector space  $E$  is (linearly) *bounded* [9, Chapter 3] if  $B$  is absorbed by every neighborhood of the origin. In what follows, unless otherwise stated,  $X$  will be a Hausdorff completely regular space and  $C_p(X)$ ,  $C_c(X)$  and  $C_b(X)$  will denote the space  $C(X)$  of all real-valued continuous functions defined on  $X$  equipped with the pointwise convergence topology, the compact-open and the compact-bounded topology, respectively. We denote by  $L(X)$  the topological dual of  $C_p(X)$  and by  $L_p(X)$  the linear space  $L(X)$  endowed with the weak\* topology.

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2010 *Mathematics Subject Classification.* 54D30, 54E15, 46A03.

*Key words and phrases.* Compact set, pointwise convergence topology, compact-bounded topology.

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